1 SYMBOLIC ANALYSIS MITACS PROJECT

within the theme of **The Mathematics of Information Technology.** [1]

2 PROJECT DESCRIPTION

The project addresses the central problems of putting mathematical analysis into the framework of symbolic computation. Our goal is to be able to deal with analytic concepts in a computational setting (specifically, in Maple) as easily as one can already do algebra. As a motivating example, consider *continuity* which is a primary concern of mathematical analysis where current methods break down, and yet it is necessary to deal with continuity in any calculation involving inequalities of functions.

The research program addresses these issues through the development and efficient implementation of algorithms for problems like exact definite integration and summation, identity and inequality verification, non-smooth differentiation, exact series evaluation and asymptotics to name just a few. Another tack we are pursuing is *reverse symbolic engineering* – the problem of determining what calculation led to a specific answer. This is a fruitful approach to a large variety of problems including the definite integration problem and the exact summation problem.

As well as this fundamental research, our program aims to make these results visible and usable to a wider community through the production of interactive mathematical tools available over the Internet and through application of our algorithms to wavelet theoretic image compression techniques.

3 PROJECT LEADER:

Name	Peter Borwein
Fields	Classical and Computational Analysis and Number Theory
Department	Mathematics and Statistics
University	Simon Fraser University
Phone Number	(604) 291-4376
Fax Number	(604) 291-4947
E-mail	pborwein@cecm.sfu.ca
Web Page	www.cecm.sfu.ca/~pborwein/

[1] KEY WORDS: Symbolic algebra, interactive mathematics, mathematical algorithms, infinite precision computational mathematics

4 OTHER TEAM MEMBERS:

4.1 SIMON FRASER UNIVERSITY:

Name	Jonathan Borwein
Field	Modern Applied Mathematics
Department	Mathematics & Statistics
University	Simon Fraser University
Phone Number	(604) 291-3070
Fax Number	(604) 291-5614
E-mail	jborwein@cecm.sfu.ca
Web Page	www.cecm.sfu.ca/ jborwein/

Name	Loki Jorgenson
Field	Visualization, Digital Publishing
Department	Adjunct – Mathematics & Statistics
University	Simon Fraser University
Phone Number	(604) 291-5616
Fax Number	(604) 291-5614
E-mail	loki@cecm.sfu.ca
Web Page	www.cecm.sfu.ca/ loki/

Name	Michael Monagan
Field	Computer Algebra
Department	Mathematics & Statistics
University	Simon Fraser University
Phone Number	(604) 291-4279
Fax Number	(604) 291-4947
E-mail	monagan@cecm.sfu.ca

Name	Petr Lisonek
Field	Computer Algebra, Combinatorics
Department	Mathematics & Statistics
University	Simon Fraser University
Fax Number	(604) 291-4947
E-mail	lisonek@cecm.sfu.ca

4.2 UNIVERSITY OF WESTERN ONTARIO:

Name	Robert Corless
Field	Applied Analysis
Department	Applied Math
University	University of Western Ontario
Phone Number	(519) 679-2111 (ext 8794)
Fax Number	(519) 661-3649
E-mail	rob.corless@uwo.ca

Name	David Jeffrey
Field	Applied Analysis
Department	Applied Math
University	University of Western Ontario
E-mail	DJJ@apmaths.uwo.ca

Name	Stephen Watt
Field	Symbolic Mathematical Computation
Department	Computer Science
University	University of Western Ontario
Phone Number	(519) 661-4244
E-mail	stephen.watt@uwo.ca.

4.3 UNIVERSITY OF GUELPH:

Name	Heinz Bauschke
Fields	Convexity, Optimization, and Functional Analysis
Department	Mathematics and Statistics
Affiliation	Okanagan UC and Univ of Guelph
Phone Number	(250) 762-5445
Fax Number	(250) 470-6004
E-mail	bauschke@cecm.sfu.ca
Web Page	www.cecm.sfu.ca/ bauschke

4.4 UNIVERSITY of QUEBEC at MONTREAL:

Name	Francois Bergeron
Field	Combinatorics
Department	Mathematics
Phone Number	(514) 987-3000
E-mail	bergeron.francois@uqam.ca

4.5 UNIVERSITY of WATERLOO:

Name	Adrian Lewis
Fields	Optimization and Variational Analysis
Department	Combinatorics and Optimization
University	University of Waterloo
Phone Number	(519) 888 4567 ext. 6983
Fax Number	(519) 725 5441
E-mail	aslewis@uwaterloo.ca
Web Page	orion.uwaterloo.ca/ aslewis

5 NON ACADEMIC PARTICIPANTS:

Company Name	Waterloo Maple Inc.
Participant	Jacques Carette
Phone Number	(519) 747-2373
Fax Number	(519) 737-1338
E-mail	jjcarette@maplesoft.com
Web Page	Maplesoft.com

Company Name	MathResources Inc.
Participant	R. Fitzgerald
Phone Number	(902) 429-1323
Fax Number	(902) 492-7101
E-mail	rfitz@istar.ca
Web Page	MathResources.com

Name	Nicholas R Swart
Company Name	Packateer
Position	Director Advanced Technology
Phone Number	(250) 717-8966
Fax Number	(250) 717-8946
E-mail	

6 SCIENCE

6.1 Project Objective

Symbolic algebra packages (Maple and Mathematica) have over the last fifteen years reached a remarkable degree of sophistication. Rather difficult problems, like exact integration of elementary functions have been significantly attacked. A number of the most important algorithms of the twentieth century like FFT's, LLL and Groebner bases are centrally incorporated. The packages can now substantially deal with large parts of the standard mathematics curriculum (and can outperform most of our undergraduates).

There is a coherent argument that they are the most significant part in a paradigm shift in how mathematics is done and certainly they have become a central research tool in many subareas of mathematics both from an exploratory and from a formal point of view. (It is acceptable now to see a line in a proof that begins "by a large calculation in Maple we see ...".) The first objective of symbolic algebra packages was to do as much exact mathematics as possible. A second objective is do it fast and to deal in an arbitrary precision environment with the more usual algorithms of analysis. Basically one would like to be able to incorporate the usual methods of numerical analysis into an exact environment or at least into an infinite precision environment.

The specific places where we wish to make progress vary with the particular expertise of current post docs and students. To date they have centered on

- modular algorithms for polynomial GCDs and resultants
- revised simplex algorithm for sparse matrices
- simplification of elementary functions (with branch cuts)
- high precision numerical quadrature
- exact solutions of ODEs
- an algebraic solver for differential (ordinary and partial) equations
- fast arbitrary precision multiplication
- integer relation algorithms and inverse symbolic computation (PSLQ)

Projects we would like to get more involved with include

- finding all zeros of arbitrary analytic functions in arbitrary regions
- visualization tools and compression algorithms
- design of specialized mathematical/engineering tool boxes
- polynomial factorization and Groebner basis computation

6.2 Sub-projects

1. Simplification: Leaders M. Monagan and P. Lisonek

The first goal of this project is to develop tools for simplifying formulae involving elementary constants and functions. This is a very important capability of a computer algebra system because the success or failure of many applications often boils down to the simplification problem. One of the main difficulties is in dealing with branch information. Hence this project includes both algebraic and analytic methods. A long term goal is to develop an organized treatment for algebraic and special functions.

This project is majority funded by MITACS funds with some additional support from NSERC operating grants of the principals and some NSERC co-funding of summer students. This is a continuation and expansion of a project started with a PIMS PDF in the year before the inception of MITACS.

2. Polynomial GCDs and factorization: Leader M. Monagan

One aspect of the simplification problem is simplifying ratios of polynomials. This usually, but not always, entails cancelling out a greatest common divisor (GCD). Another aspect is factoring a polynomial as the factorization will usually simplify a problem. The goal of this project is to design and implement efficient methods for solving the polynomial GCD problem, the related problem of computing polynomial resultants, and the polynomial factorization problem over the integers, finite fields, algebraic number and function fields. Polynomial GCD, resultant, and factorization computations occur in many places in a computer algebra system. Efficient solutions to these problems is of central importance to their overall capability.

This is a new project since the inception of the NCE. It is majority funded by MITACS funds with some additional support from NSERC operating grants of the principals and some NSERC co-funding of summer students.

3. Symbolic analysis of functions: Leader M. Monagan

The purpose is to develop tools and algorithms to provide an analysis of a function represented by a symbolic formula, a program, or a differential equation. We are also interested in determining the asymptotic behavior of a program and the validity of the derivatives of a program computed by automatic differentiation.

By way of example we describe one application requiring an analysis of a function represented explicitly by a formula. The following function, from an application in Geophysics,

$$E(\gamma) = -\frac{(1+2\,\gamma)^2\,\beta}{r^2\gamma}\sqrt{{H_2}^2+\frac{r^2}{1+2\,\gamma}}\left(\frac{\sqrt{(X-r)^2+{H_1}^2}}{V_1}+\frac{\sqrt{{H_2}^2+\frac{r^2}{1+2\,\gamma}}}{\beta}\right)$$

measures the sensitivity of a function of γ to measurement errors in the travel time of a signal through a two-layered medium in which the top layer is isotropic and the bottom layer is weakly anisotropic. Given the following information, $0 < r < X$, $\beta > 0$, $V_1 > 0$, $H_1 > 0$, $H_2 > 0$, and $|\gamma| << 1$, we seek to determine which factors contribute to a large magnitude of E(γ). From a given physical problem where β, V_1, H_1 and γ are determined, what are the best choices for X and H_1 to minimize the magnitude of E(γ)? Problems like this are routinely handled by applied mathematicians using ad hoc techniques. We seek to automate the analysis. The tools required include a treatment of inequalities and relations.

This is a new project since the inception of the NCE. It is exclusively funded by MITACS funds.

4. Exact solutions of differential equations: Leaders M. Monagan and E. Cheb-Terrab

This project aims at finding new algorithms for finding exact solutions to ODEs, PDEs and systems of them. The project includes implementing the algorithms in a Computer Algebra System (CAS) - we choose Maple.

The determination of exact solutions to differential equations is a challenge. Few algorithms are known, and they work mostly for linear equations. For the non-linear case, the methods based on finding point symmetries or restricted forms of integration factors cover a quite small portion of the problem (albeit, this portion is the one we see being used in modelling, most probably because the related differential equations are actually the only ones we know how to solve exactly). The main motivation here is that, having more general exact solution algorithms and implemented in a user-friendly CAS environment, will remove some of the present barriers researchers have when modelling real and theoretical problems.

Concerning the techniques involved in this project, the main ones are: symmetries, integrating factors, formulation of equivalence problems for the non-linear case and determination of (as general as possible) hypergeometric solutions for the linear case. All these methods are considered in the framework of both ODEs and PDEs.

This is a new project since the inception of the NCE. It is exclusively funded by MITACS.

5. Reverse symbolic engineering and Interactive network mathematics: Leader P. Borwein

This project is pursued in conjunction with the a project of the Telelearning NCE. It involves building an interactive collection of algorithms (with an interface) to solve a number of "reverse engineering" problems in analysis. For example, given a sequence how is it generated? Or given a real number by its decimal expansion when does it have an identifiable closed form? Some of this involves tieing together extant tools like GFUN or Sloane's "Handbook of Integer Sequences". Some of the project involves building a much improved version of our own "inverse symbolic calculator".

Issues related to the activation, interaction and delivery of mathematics for computational user/researchers are explored. Existing and next-generation technologies will be integrated into context-based prototypes. Technologies like OpenMath, XML, Java and JavaBeans, as well as established digital publishing and network communications approaches will be applied.

This project has complementary interests for related groups like MathResources Inc., Maple Waterloo Inc., the Canadian Mathematical Society, and projects within the NCE Telelearning.

Much present activity revolves around establishing the recently CFI funded Simon Fraser High Performance Computing Centre which will offer the most sophisticated Canadian resources for parallel symbolic computation.

Central to this problem is a careful comparison of the competing methods such as PSLQ and LLL.

This is a new project since the inception of the NCE. It is about 75% funded by MITACS and about 25% funded by NCE funds through the Telelearning NCE.

6. Wavelet based compression and related problems concerning quality assessment of images: Leader J. Borwein

The new topic of compression is being undertaken with Mason Macklem and Heinz Bauschke. Mason is a newly admitted Masters student working with Packeteers of Canada (a Kelowna company) on wavelet-based and other image compression and reconstruction problems such as:

- calculable measures for assessing image quality — a vexing issue here and in medical image reconstruction;
- accelerated compression of images similar to previously compressed images;
- competitive compression schemes where the decode engine is computationally very light (e.g. for mobile platforms such as the Cassiopeia for which MathResources has been building mathematical software).

This is a brand new project exclusively funded out of MITACS funds.

7. Computer algebra tools for analysis: Leader R. Corless

This project considers algorithm development and implementations for analytic problems of mathematics.

The software issues are the construction of semantically aware Web display and interoperability tools [using MathML, OpenMath, Maple, and Aldor] so that, for example, analytic answers may be intelligently displayed and evaluated. This project aims at correct but efficient solution of symbolic problems on specialization.

MITACS has partially funded a post doctoral fellow at UWO on this project. The other funds are provided by researchers at UWO.

8. Exact solutions of differential equations, series solutions and generalizations: Leader F. Bergeron

The goal of this project is to get exact solutions of differential equations, functional equations and more general equations such as q-equations. From a theoretical perspective, only special cases of these general problems are tractable, and even the case of finding algebraic solutions of linear equations of small degree is difficult (see: M.F. Singer and F. Ulmer, J. Symbolic Computation, 1993). A different approach is pursued, one that relies in part on a "reverse symbolic engineering" outlook, but also on exact manipulation of generating functions and the equations they satisfy.

One powerful and fast technique is to begin by "guessing" an algebraic candidate for a solution of a given differential equation, and use the algebraic equation satisfied by this candidate to give an "automatic" proof that it does satisfy the desired equation.

MITACS has funded graduate students at UQAM to work on this project.

7 PROGRESS MADE and RESULTS OBTAINED

7.1 Summarize the Project's main achievements.

The project is up and running very satisfactorily. We have made significant progress towards the specific projects as well as serendipitous progress in other directions. We have developed and incorporated significant Maple related code (as outlined below). This involves both theoretical developments and implementation improvements. We have substantially prototyped a reverse engineering tool "RevEng". We have forged a new partnership with Packateer and become involved with compression algorithms. The principal achievements are listed in the next two sections.

7.2 Other Highlights:

1. The construction of Maple's PDE system solver (program) has had its functionality greatly improved by E. Cheb-Terrab. (Neither Maple nor Mathematica nor any of the other general purpose Computer Algebra Systems are distributed with such functionality.) A first version of the program was integrated recently to the Maple research library, and hence, will be distributed in the next version of Maple. Examples of the new PDE solver are illustrated in the appendix.

   This resulted in a speed up of the numerical solving of ODEs by a remarkable factor of 30; that is, performing the numerical solving of ODEs inside Maple but (almost) at the speed of a C numerical routine.

2. P. Borwein, P. Lisonek and C. Percival have found the first new solutions of size 10 on a very old problem called the Tarry-Escott problem. This involved both theoretical developments and a very large scale parallel computation using several hundred lab PCs at night.

3. C. Percival, using quite remarkable algorithms of Borwein, Bailey and Plouffe for computation of individual bits of poly logarithmic constants (like Pi) implemented and orchestrated a highly distributed and very elegant web based computation of the quadrillionth bit of Pi. The calculation took a total of about 1.2 million cpu hours, and was done using 'idle' time slices (cpu time which no other program is using) on systems running windows 95, 98, NT, 2000, and Linux .

4. C. Percival gives results that show that it is now practical to use floating-point FFTs for large integer multiplication, which generally gives a significant (factor of 2 at least) improvement over finite field FFTs

5. A. Solomon (with A. Cutting) has done some of the first work in semigroups with rational cross-section. This gives decidability and complexity results for word problems in semigroups and therefore all associative algebras. Results were applied to obtain an intractability theorem for certain classes of inverse semigroups which are important in physics and geometry.

6. A. Solomon has written a Java API supporting Internet Accessible Mathematical Computation. This will be released in November together with demos using it written at St. Andrews (Solomon, Linton, Struble) and SFU (Solomon, Cooper).

7.3 IMPEDIMENTS:

- The final IP arrangements have not been made with Maple. Though at the time of writing this report we are about to sign. This complicates the issue of incorporating code in Maple. Only two weeks ago we received a copy of the development version of Maple which makes it possible for us to start installing code.
- The postdoc hire at UWO was delayed because the prospective hire 'un-accepted' the offer, for personal reasons (cancer in the family).

8 RESULTS of the RESEARCH

PAPERS:

Peer-reviewed publications (appeared or accepted):

1. F. Bergeron and F. Gacon "Counting Young Tableaux of Bounded Height" Journal of Integer Sequences, Volume 3 (2000), Article 00.1.7.
2. F. Bergeron, A. M. Garsia, M. Haiman and G. Tesler "Identities and Positivity Conjectures for Some Remarkable Operators in the Theory of Symmetric Functions" Methods and Applications of Analysis, Volume 6, No. 3, (1999), 363-420.
3. David Borwein and Jonathan M. Borwein, "Some remarkable properties of sinc and related integrals," *The Ramanujan Journal*, [Accepted June 2000.] .
4. Jonathan M. Borwein, David J. Broadhurst and Joel Kamnitzer, "Central Binomial Sums, Multiple Clausen Values, and Zeta Values," *Experimental Mathematics*, [Accepted March 2000.].
5. J. M. Borwein, D. M. Bradley, D. J. Broadhurst, P. Lisonek, "Special values of multiple polylogarithms." Transactions of the American Mathematical Society, accepted.
6. J. M. Borwein and Robert M. Corless, "Emerging tools for experimental mathematics" American Mathematical Monthly, December 1999. [2]
7. P. Borwein and K. Hare "Some Computations on the Spectra of Pisot and Salem Numbers" Math Comp, To Appear

[2] These papers take advantage of EZFace Java tool we developed, which is at the URL http://www.cecm.sfu.ca/projects/ezface+

8. Robert M. Corless and Stephen M. Watt, "Software Tools for Mathematical Communication", in Video Proceedings of the Future of Mathematical Communications, MSRI, Berkeley. Maple worksheet and videos available at http://www.msri.org/publications/video

9. Robert M. Corless, James H. Davenport, David J. Jeffrey, and Stephen M. Watt, "According to Abramowitz and Stegun", Sigsam Bulletin, volume 34, no. 2, June 2000.

10. Robert M. Corless, James H. Davenport, David J. Jeffrey, Gurjeet Litt, and Stephen M. Watt, "Reasoning about the Elementary Functions of Complex Analysis", Proceedings AISC 2000, Madrid. Springer Lecture Notes in Artificial Intelligence 1930, to appear. [Journal version planned, with R. J. Bradford]

11. E.S. Cheb-Terrab and A.D. Roche, *"Abel Equations: Equivalence and New Integrable Cases"*, Computer Physics Communications, 130 p.197 (2000).

12. E.S. Cheb-Terrab and T. Kolokolnikov, "Solving first order ODEs using linear transformations", accepted for *Seventh Rhine Workshop on Computer Algebra* (**RWCA/00**), Bregenz, Austria, March 2000.

13. E.S. Cheb-Terrab and A.D. Roche, *An Abel ODE class generalizing known integrable classes*, accepted for publication in the European Journal of Applied Mathematics.

14. R. Israel, P. Lisonek, Metric invariants of tetrahedra via polynomial elimination. *Proceedings of the International Symposium on Symbolic and Algebraic Computation 2000, (Ed. C. Traverso)* ACM Press pp. 217–219, 2000.

15. E. Kaltofen. "Challenges of symbolic computation: my favorite open problems". J. Symbolic Comput., 29(6):891-919, 2000. With an additional open problem by Robert M. Corless and David J. Jeffrey.

16. Gurjeet Litt, Unwinding numbers for elementary functions. MSc Project, Dept Applied Math, U. Western Ontario, Dec 1999. (Supervisor: Robert M. Corless)

17. M.B. Monagan, A.D. Wittkopf, On the Design and Implementation of Brown's Algorithm over the Integers and Number Fields. *Proceedings of the International Symposium on Symbolic and Algebraic Computation 1999* ACM Press, pp. 225–233, 2000.

18. A. Cutting and A. Solomon, "Remarks concerning finitely generated semigroups having regular sets of unique normal forms" (to appear in J. Aus. Math. Soc. series A).

Other publications (including submissions):

1. J. M. Borwein and T. Stanway ”Numerical and computational mathematics (at the undergraduate level),” Proceedings of the Technology in Mathematics Education at the Secondary and Tertiary Levels Conference, June 1999 at Brock University, August 1999. (Also to appear in Cubo.)
2. Peter Borwein and Loki Jorgenson ”Visible Structures in Number Theory” Submitted, Sept, 2000
3. E.S. Cheb-Terrab and T. Kolokolnikov, “First order ODEs, Symmetries and Linear Transformations”; submitted (Jul/00) to European Journal of Applied Mathematics.
4. E.S. Cheb-Terrab and A.D. Roche ”An Abel ODE Class Generalizing Known Integrable Classes” submitted to European Journal of Applied Mathematics, 1999
5. E.S. Cheb-Terrab and A.Wittkopf, ”Fast Numerical Solution of ODEs in a Symbolic Environment” In preparation for Computer Physics Communications
6. Robert M. Corless, David J. Jeffrey, and David E. G. Hare, “Maple has a signed zero” ECCAD 2000, London.
7. Gurjeet Litt and Robert M. Corless, “Unwinding numbers for elementary functions”, ECCAD 2000, London.
8. J. Mulholland, M.B. Monagan, Trigonometric Polynomials: Factorization in the ring $Q[s,c]/ < s^2 + c^2 - 1 >$. To be submitted for publication in the proceedings of ISSAC 2001.
9. E. Kaltofen, M.B. Monagan, A.D. Wittkopf, On the Modular Polynomial GCD Algorithm over the Integers, Finite Fields, Number Fields, and Euclidean Rings. To be submitted to the Journal of Symbolic Computation.
10. Colin Percival ”Rapid multiplication modulo the sum and difference of highly composite numbers” submitted to Math. Comp.
11. A. Solomon, C. A. Struble, A. Cooper and S. A. Linton, “The JavaMath API – A framework for internet accessible mathematical services”, submitted to Journal of Symbolic Computation.

9 OTHER RESEARCH PERSONNEL

9.1 Visiting Academics:

Name	David Bailey
University	Livermore Labs (Berkeley)
Funded by NCE	
Funded Non-NCE	100%

Visiting SFU from Livermore Labs (Berkeley) August, 1999.
Lecture: "Parallel Integer Relation Detection: Techniques and Applications"

Name	Francois Bergeron
University	University of Quebec
Funded by NCE	
Funded Non-NCE	100%

Visiting SFU July-August, 1999

Name	Frederic Chyzak
University	INRIA
Funded by NCE	25%
Funded Non-NCE	75%

Visiting UQAM from October 25, 99 - November 5, 1999

Name	Michael Lamoureux
University	University of Calgary
Funded by NCE	
Funded Non-NCE	100 %

Visiting SFU July-August, 1999

Name	Kieth Geddes
University	University of Waterloo
Funded by NCE	
Funded Non-NCE	100 %

Visiting SFU Jan-April, 2000

9.2 POST-DOCTORAL FELLOWS:

Name	Janez Ales
University	SFU
Funded by NCE	100%
Funded Non-NCE	
Period of involvement	January 2000 –

Name	Petr Lisonek
University	SFU
Funded by NCE	80%
Funded Non-NCE	20%
Period of involvement	January 1999 – September 2000

Name	Edgardo Cheb-Terrab
University	SFU
Funded by NCE	20%
Funded Non-NCE	100%
Period of involvement	February 1999 –

Name	Ron Ferguson
University	SFU
Funded by NCE	50%
Funded Non-NCE	0%
Period of involvement	September 2000 –

Name	Matt Klassen
University	SFU
Funded by NCE	100%
Funded Non-NCE	0%
Period of involvement	January 1999 – September, 1999

Name	Andrew Solomon
University	SFU
Funded by NCE	70%
Funded Non-NCE	30%
Period of involvement	April 2000 –

J. Ales and A. Solomon are full time on this project. R. Ferguson is 1/2 time (with on other commitments at SFU). E. Cheb Terrab is a now a full time employee of Maple Inc in Vancouver. We are continuing one research project with him.

9.3 RESEARCH STAFF:

Name	Alan Cooper
University	SFU
Funded by NCE	40% (with Telelearning NCE)
Funded Telelearning-NCE	60%
Period of involvement	July 2000 –

Name	Stephen Funk
University	SFU
Funded by NCE	50%
Funded Non-NCE	50%
Period of involvement	July 1999 – April 2000

A. Cooper is part of a joint project with the telelearning NCE. Currently he is funded by telelearning. He will move to MITACS funding in November.

9.4 GRADUATE STUDENTS:

Name	Hans Bauck (M.Sc. candidate)
University	SFU
Funded by NCE	15%
Funded Non-NCE	85%
Period of involvement	February 1999 –

Name	Mhenni Benghorpal (Ph.D. candidate)
University	Western Ontario
Funded by NCE	50%
Funded non-NCE	50%
Period of involvement	Feb 2000 – April 2000

Name	Jennifer de Kleine (M.Sc. candidate)
University	SFU
Funded by NCE	15%
Funded non-NCE	85%
Period of involvement	September 1999 –

Name	Gurjeet Litt (M.Sc. candidate, soon to be Ph.D. candidate)
University	Western Ontario
Funded by NCE	50%
Funded non-NCE	50%
Period of involvement	May 1999 – December 1999

Name	Mason Macklem (M.Sc. candidate)
University	SFU
Funded by NCE	0%
Funded non-NCE	100%
Period of involvement	September 1999 –

Name	Alan Meichsner (M.Sc. candidate)
University	SFU
Funded by NCE	20%
Funded non-NCE	80%
Period of involvement	September 1999 –

Name	Francis Gascon
University	UQAM
Funded by NCE	75%
Funded non-NCE	25%
Period of involvement	June 99 December 99

Name	Kevin Hare (Ph.D. candidate)
University	SFU
Funded by NCE	0%
Funded Non-NCE	100%
Period of involvement	February 1999 –

Name	Terry Stanway (M.Sc. candidate)
University	SFU
Funded by NCE	
Funded Non-NCE	100%
Period of involvement	February 1999 –

Name	Allan Wittkopf (Ph.D. candidate)
University	SFU
Funded by NCE	
Funded Non-NCE	100%
Period of involvement	February 1999 –

Mason Macklem is the only student 100% funded by our project (all industrial funds). The other students get partial stipends at various times.

9.5 OTHERS

(e.g. co-op student, summer student, consultant, etc.)

Name	Anouk Brlek
University	UQAM
Funded by NCE	25%
Funded Non-NCE	75%
Period of involvement	June 99 – September 99

Name	Jamie Mulholland (Summer NSERC)
University	SFU
Funded by NCE	00%
Funded Non-NCE	100%
Period of involvement	Summer 2000 and Fall 2000 as a Masters student at UBC

Name	Craig Pastro (Summer NSERC)
University	SFU
Funded by NCE	40%
Funded Non-NCE	60%
Period of involvement	Summer, 2000

Name	Colin Percival (Summer NSERC)
University	SFU
Funded by NCE	40%
Funded Non-NCE	60%
Period of involvement	Summer, 1999 and Summer, 2000

Name	Simon Plouffe
University	UQAM
Funded by NCE	40%
Funded Non-NCE	60%
Period of involvement	September 1999 –