

Decagonal Tilings in Medieval Islamic Architecture



Outline



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 - Plane Tilings
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 - Periodic Tilings

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 - Periodic Tilings
- ▶ Aperiodic Tilings
 - Penrose Tilings
 - Girih Tilings

Plane Tilings



Plane Tilings

- ▶ Collection of plane figures



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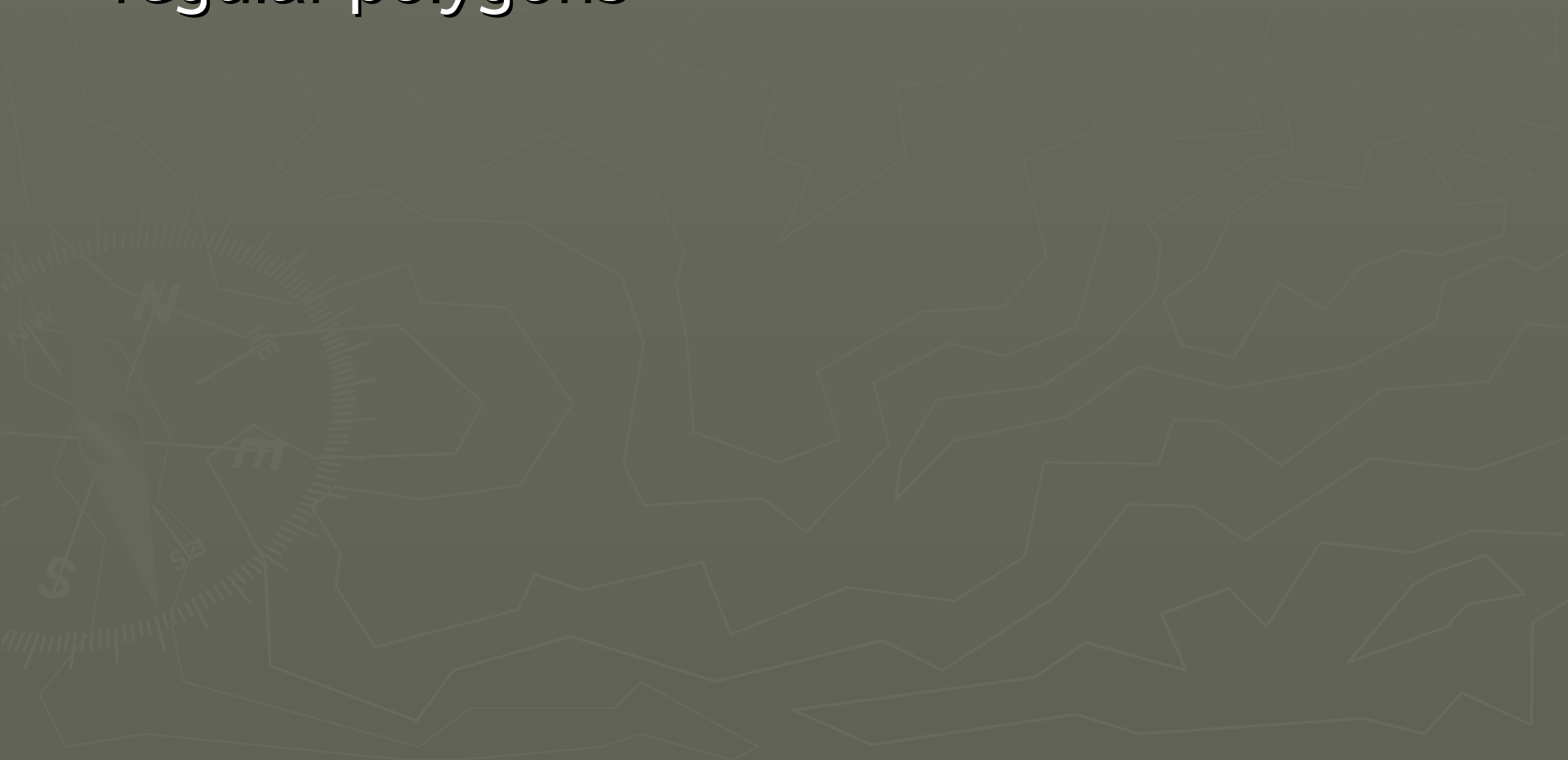


Regular Tilings



Regular Tilings

- ▶ Plane tiling of congruent regular polygons

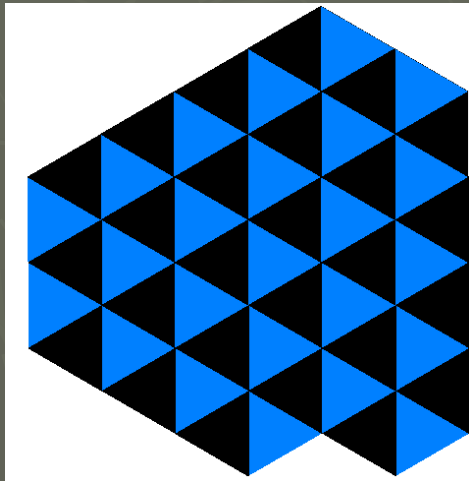


Regular Tilings

- ▶ Plane tiling of congruent regular polygons
- ▶ Only three exist

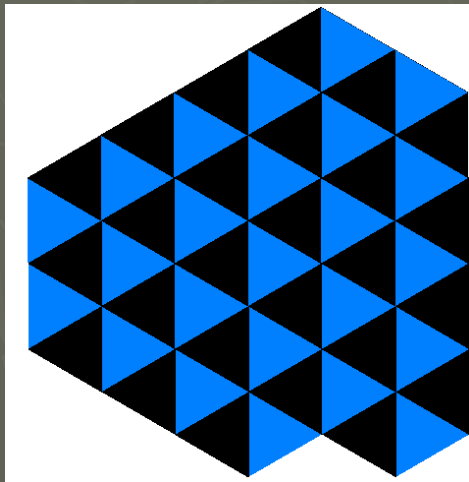
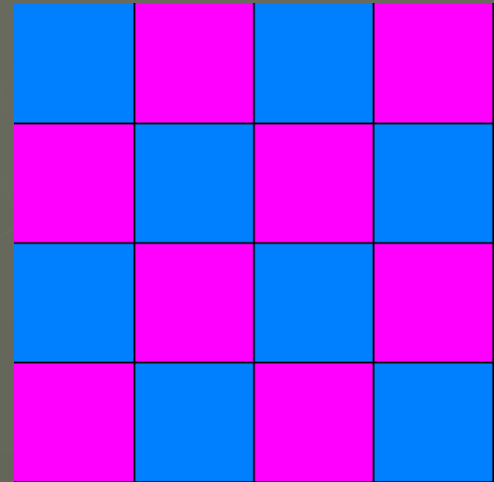
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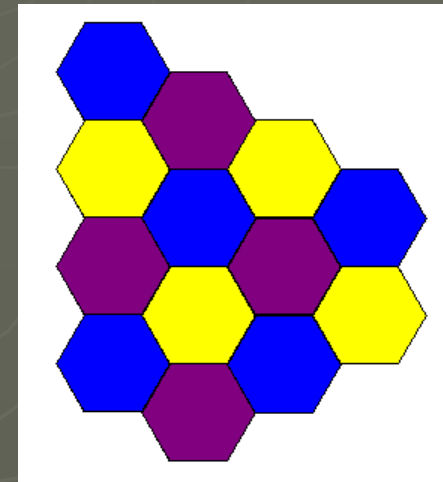
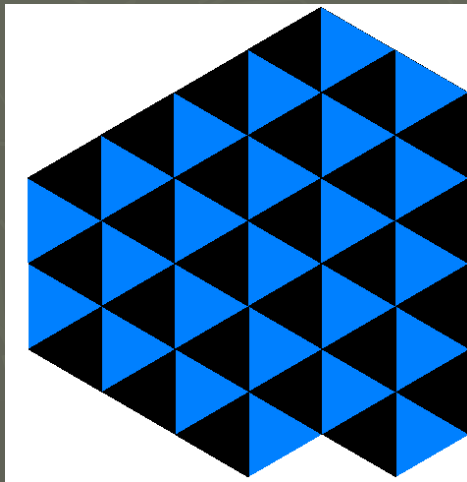
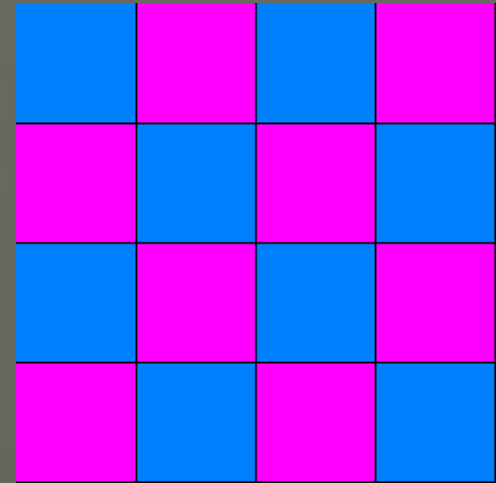
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Periodic Tilings



Periodic Tilings

- ▶ Tiling of the plane with translational symmetry



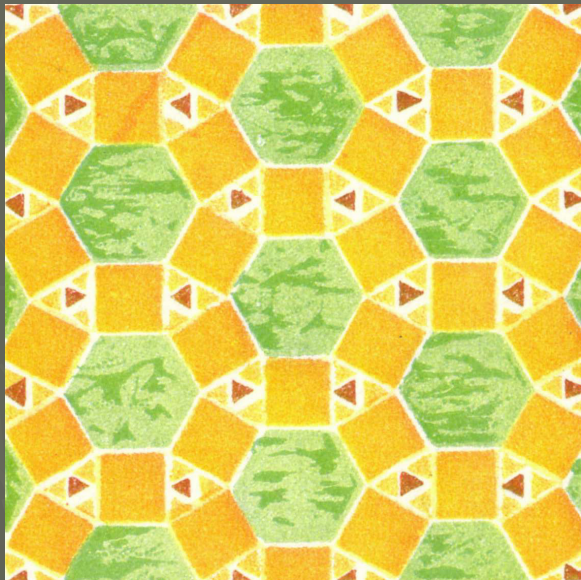
Periodic Tilings

- ▶ Tiling of the plane with translational symmetry
- ▶ Only 2-fold, 3-fold, 4-fold and 6-fold rotational symmetry is allowed



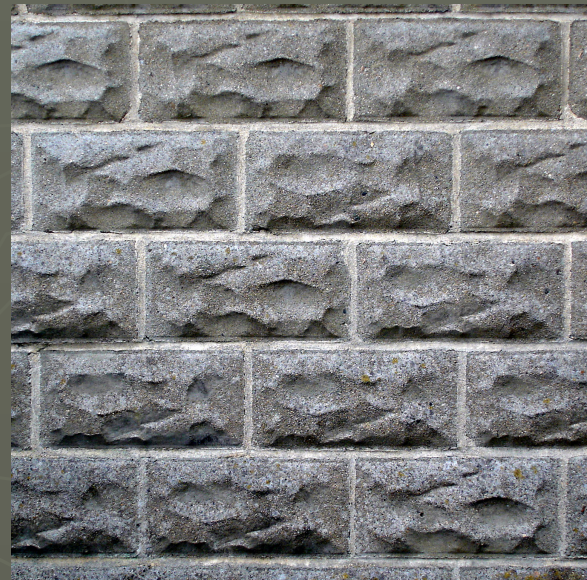
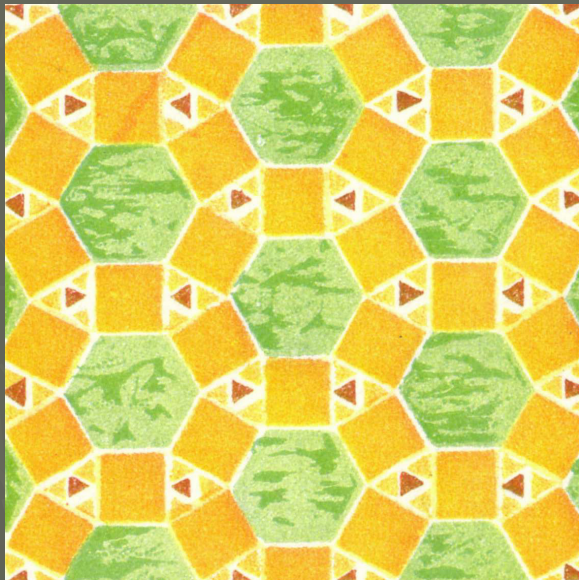
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Penrose Tiling

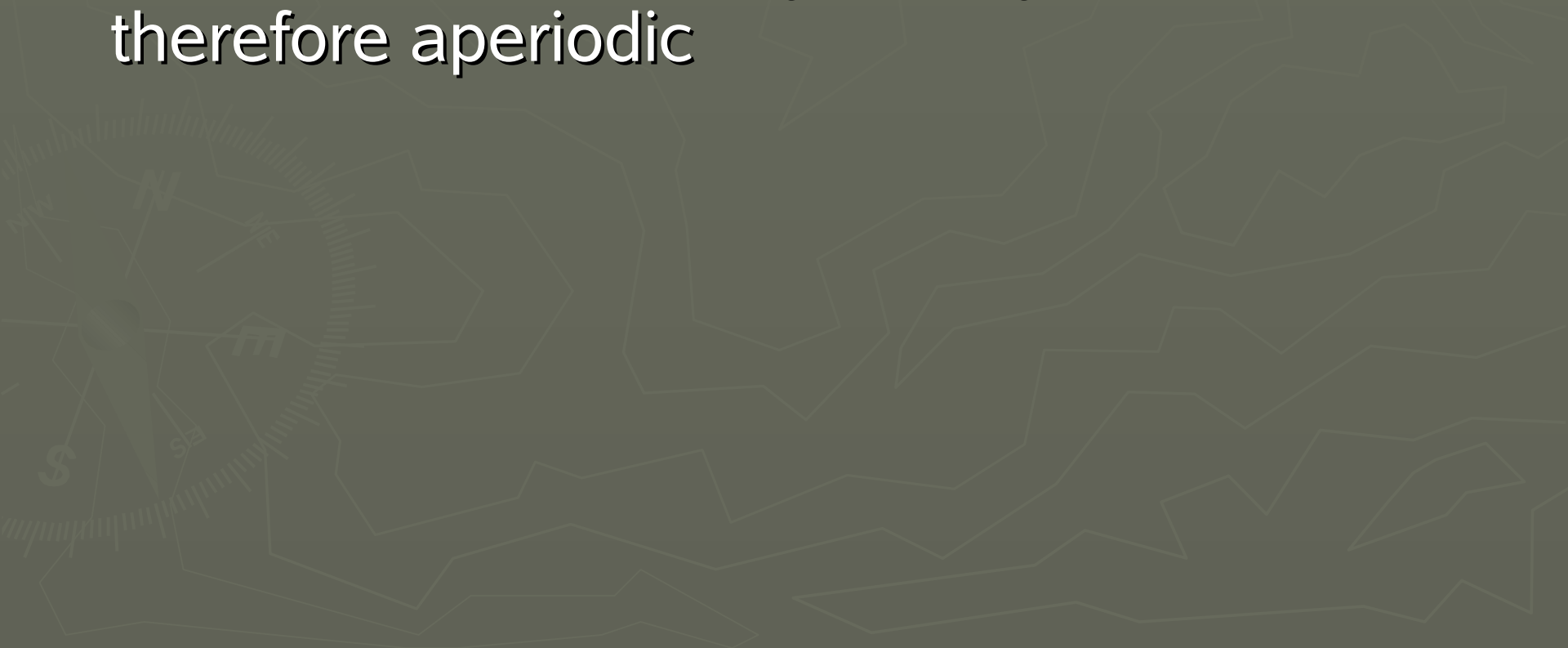


Penrose Tiling

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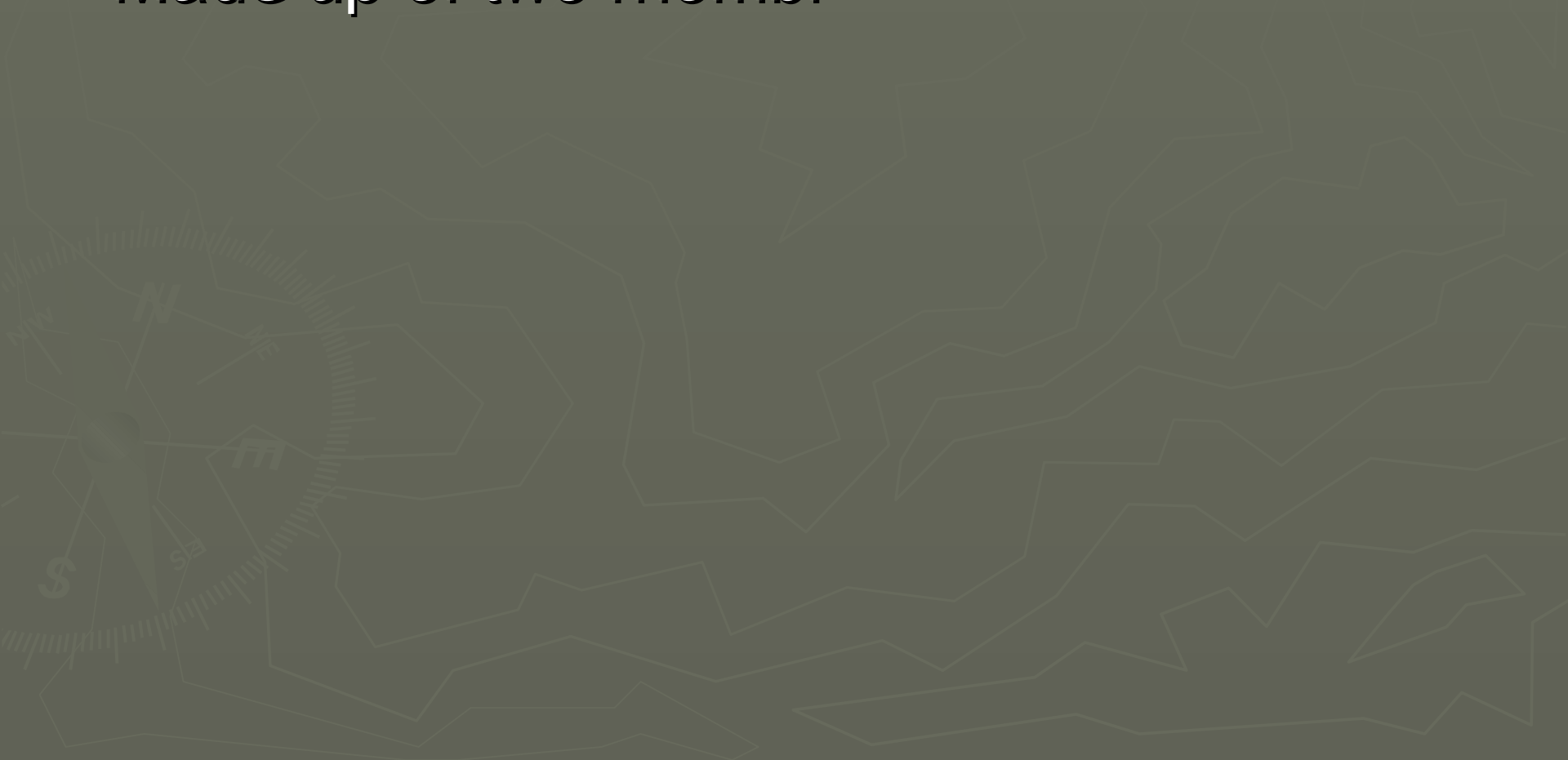


Penrose Tiling

- ▶ Penrose tiling was developed by Roger Penrose in 1973
- ▶ Has no translational symmetry and is therefore aperiodic
- ▶ Is quasi-periodic (in physics this property is called quasi-crystalline)

Penrose Tiling

- ▶ Made up of two rhombi



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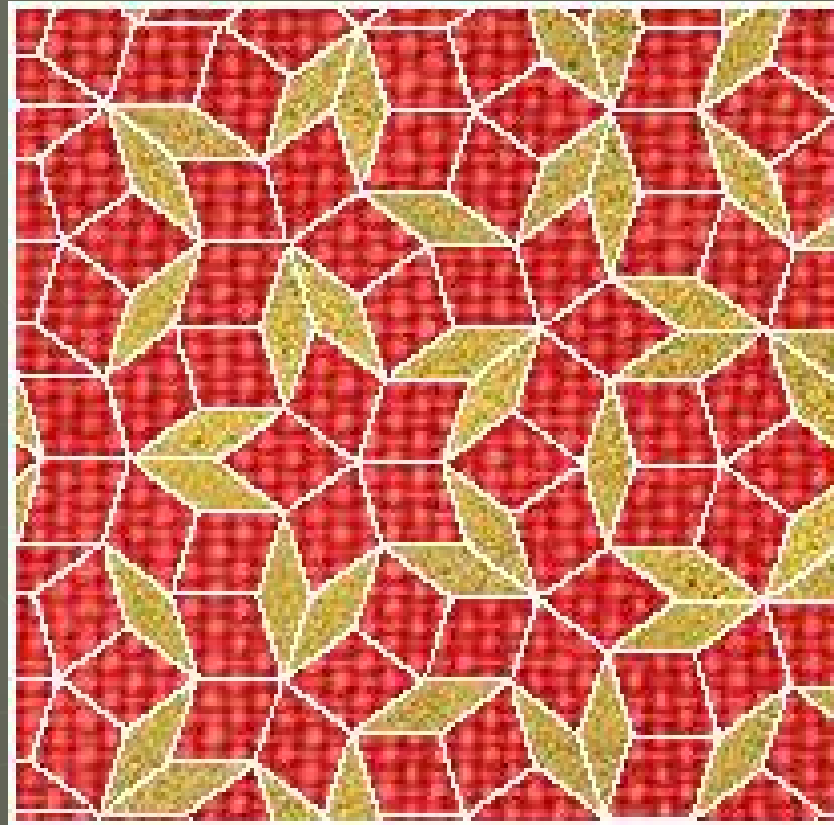
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 - $\{72, 108, 72, 108\}$ degrees
 - $\{36, 144, 36, 144\}$ degrees
- ▶ Only one rule: no two adjacent tiles can form a parallelogram
- ▶ Ratio between number of each tiles is golden ratio

Penrose Tiling



Girih Tiling



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Girih Tiling

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Girih Tiling

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- ▶ Locally display 10-fold rotational symmetry, and therefore cannot be periodic

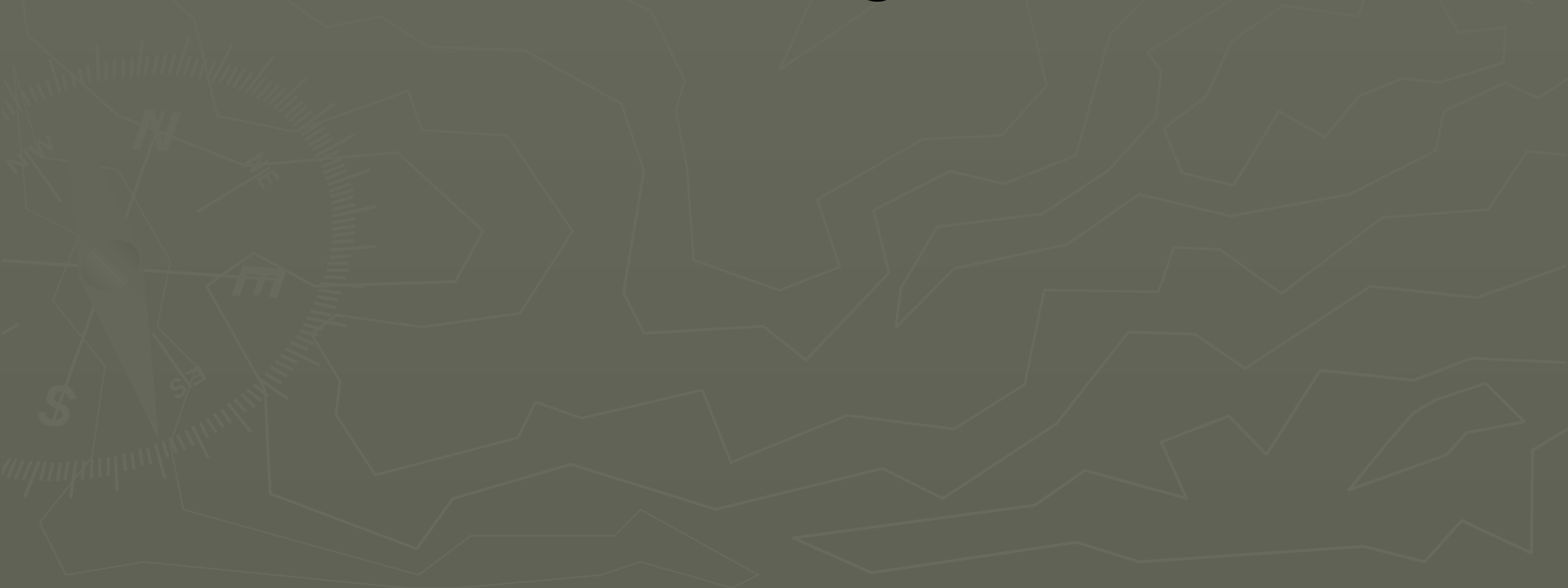


Girih Tiling

- ▶ Very complex patterns that appear throughout Islamic art and architecture
- ▶ Locally display 10-fold rotational symmetry, and therefore cannot be periodic
- ▶ Initially thought to have been created using compass and straight edge method

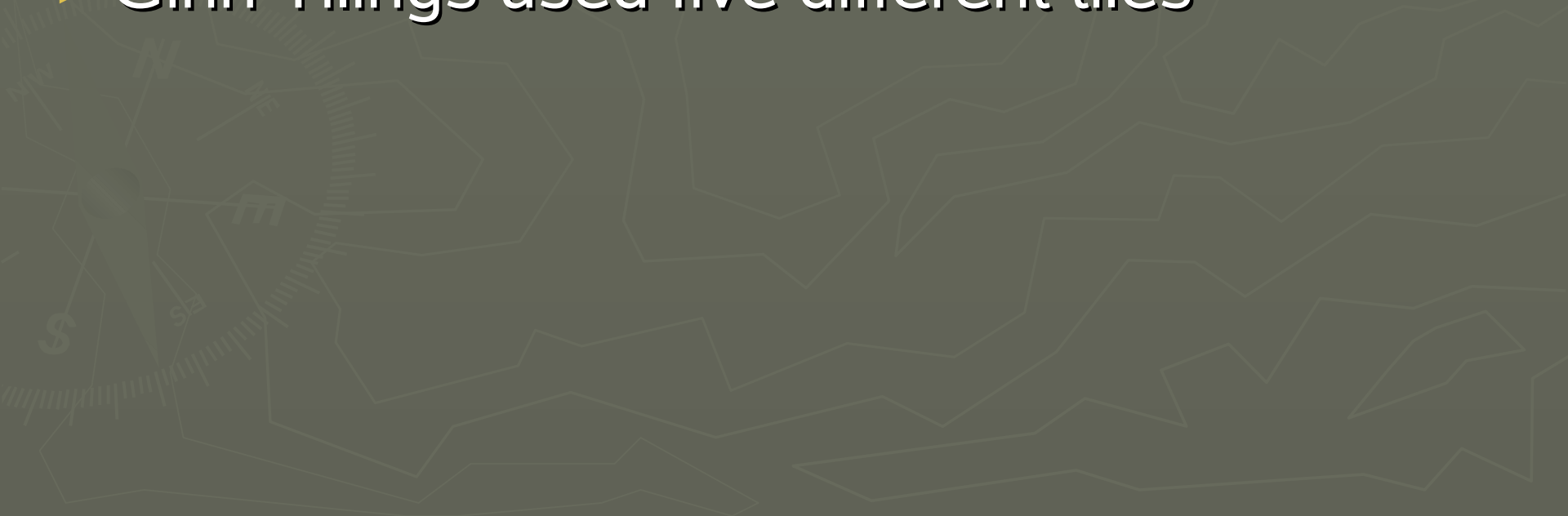
Girih Tiling

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Girih Tiling

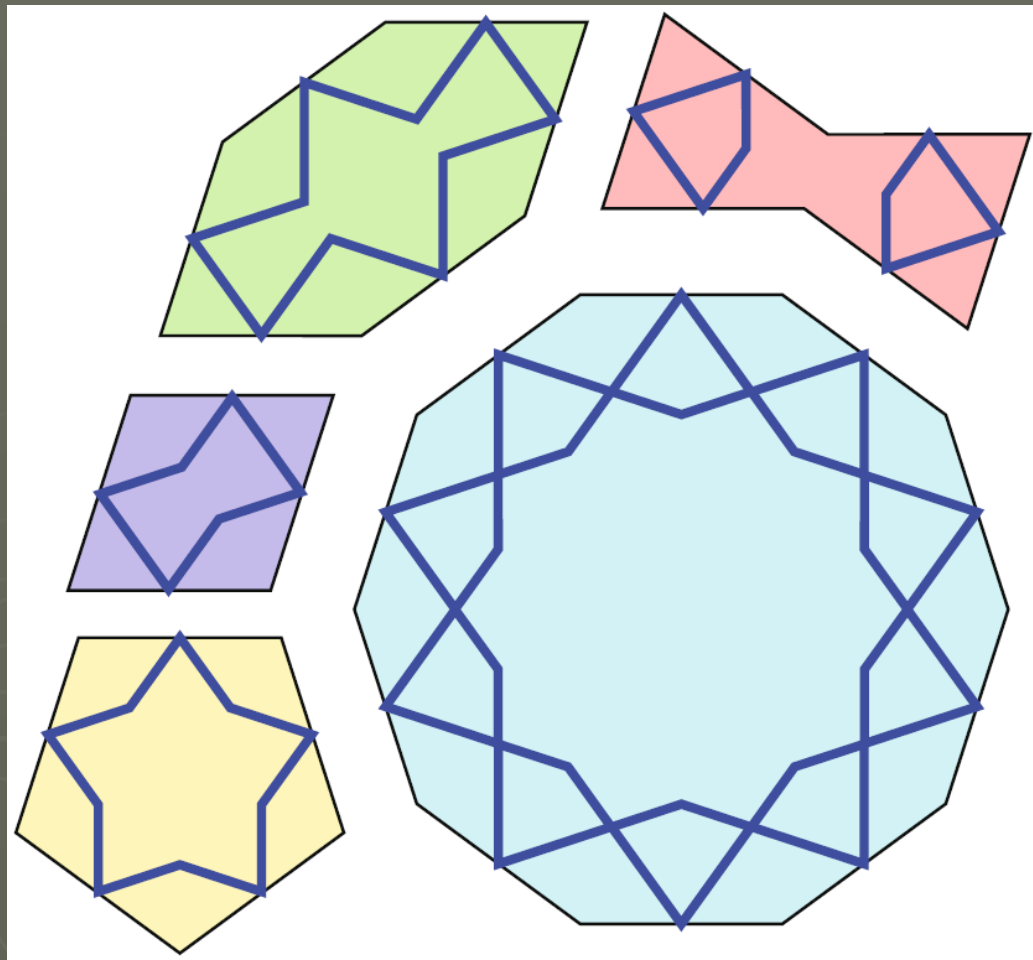
- ▶ Peter J. Lu at Harvard and Paul J. Steinhardt at Princeton found that Girih Tilings exhibit advanced decagonal quasicrystal geometry like that of Penrose Tilings
- ▶ Girih Tilings used five different tiles



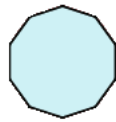
Girih Tiling

- ▶ Peter J. Lu at Harvard and Paul J. Steinhardt at Princeton found that Girih Tilings exhibit advanced decagonal quasicrystal geometry like that of Penrose Tilings
- ▶ Girih Tilings used five different tiles
- ▶ Can be mapped to Penrose Tilings

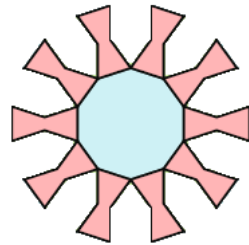
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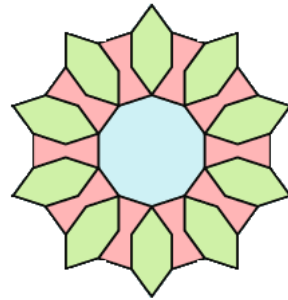
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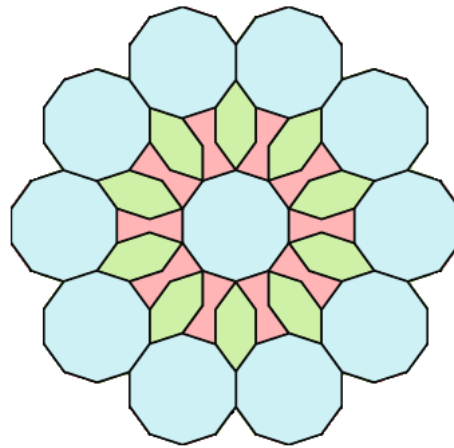
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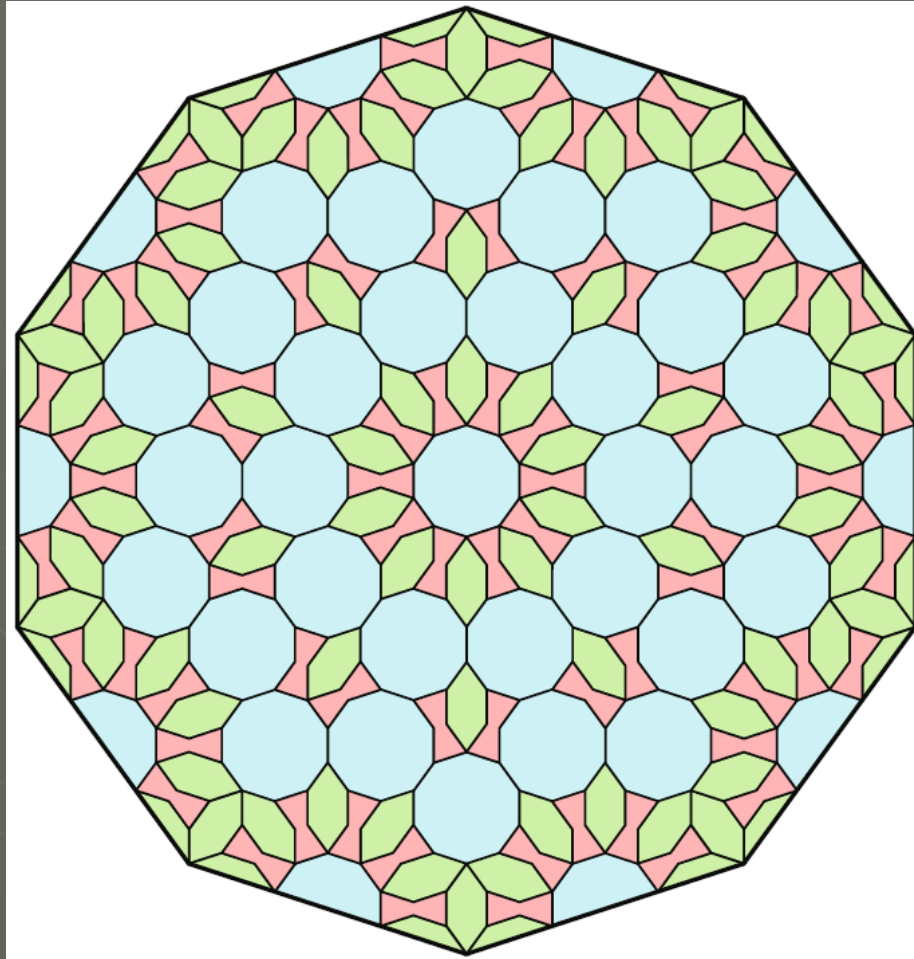
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Examples

<http://www.sciencemag.org/cgi/content/full/315/5815/1106/DC1c>

References

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2. **Category: Periodic Tiling Image.** Wikipedia.org, http://en.wikipedia.org/wiki/Category:Periodic_tiling_image
3. **Penrose Tilings.** Science U, <http://www.scienceu.com/geometry/articles/tiling/penrose.html>