How to share a Pizza

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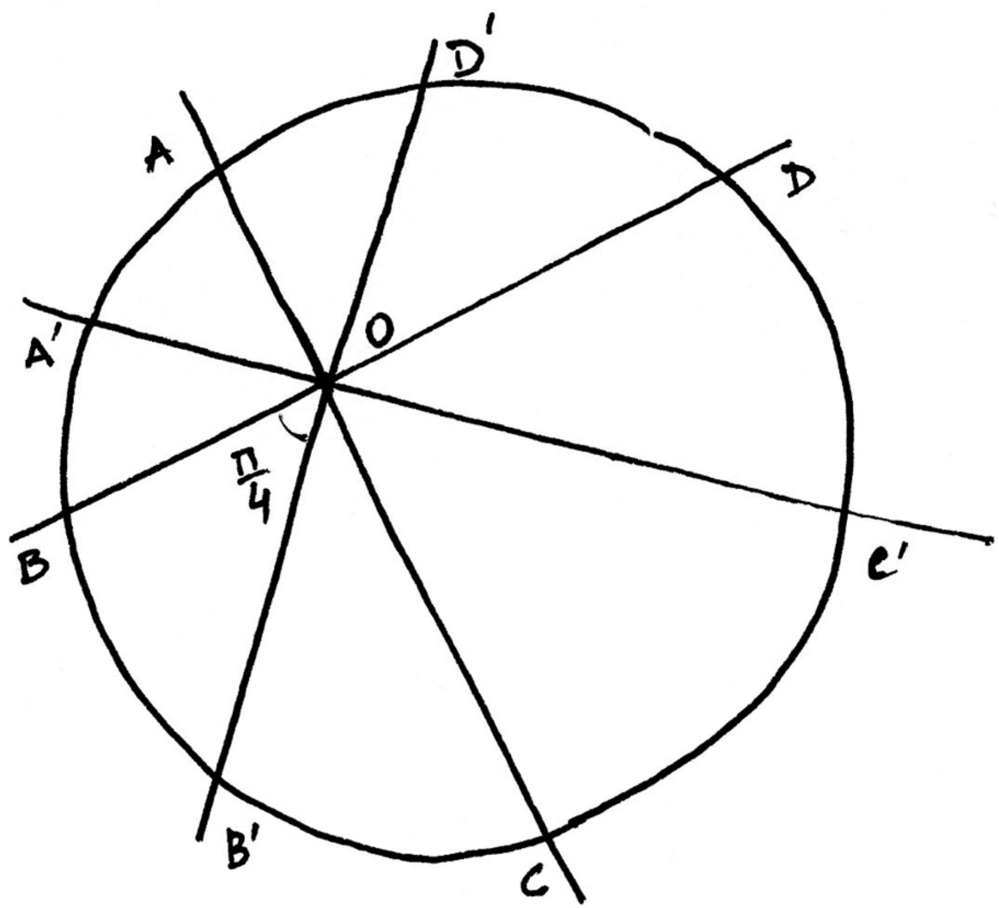
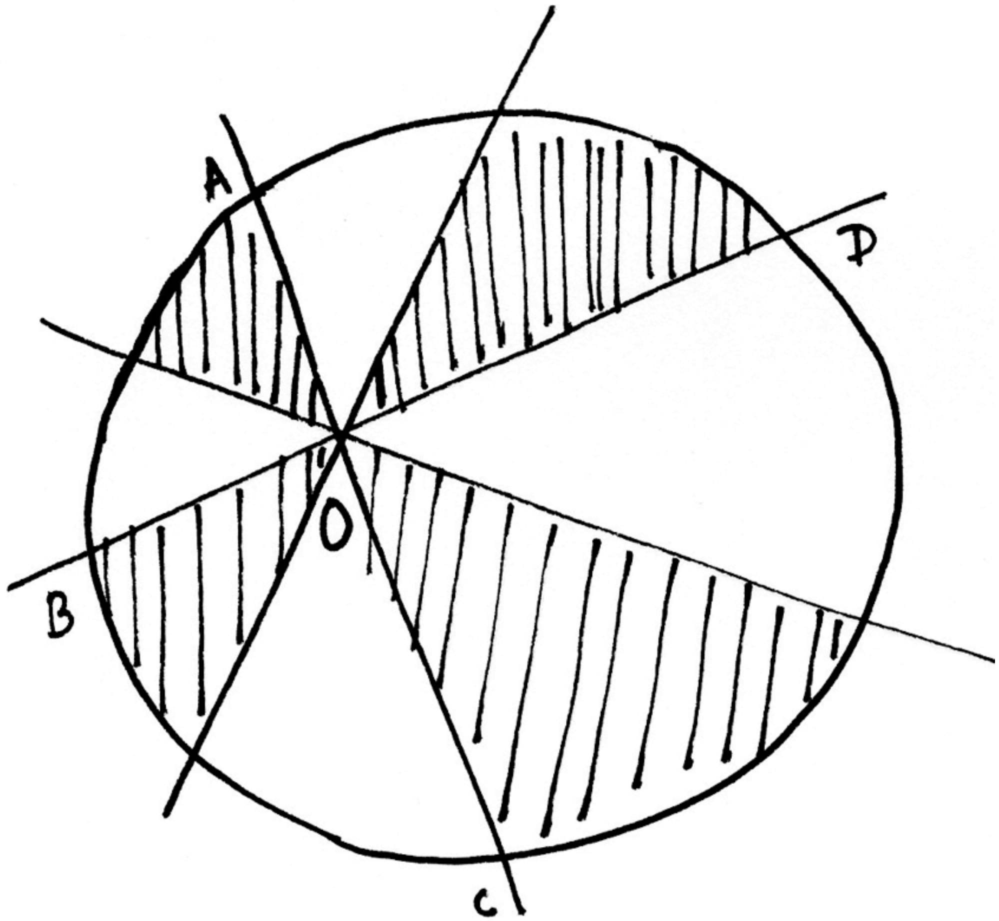

D'
A
D
O
A'
π/4
B
C'
B'
C

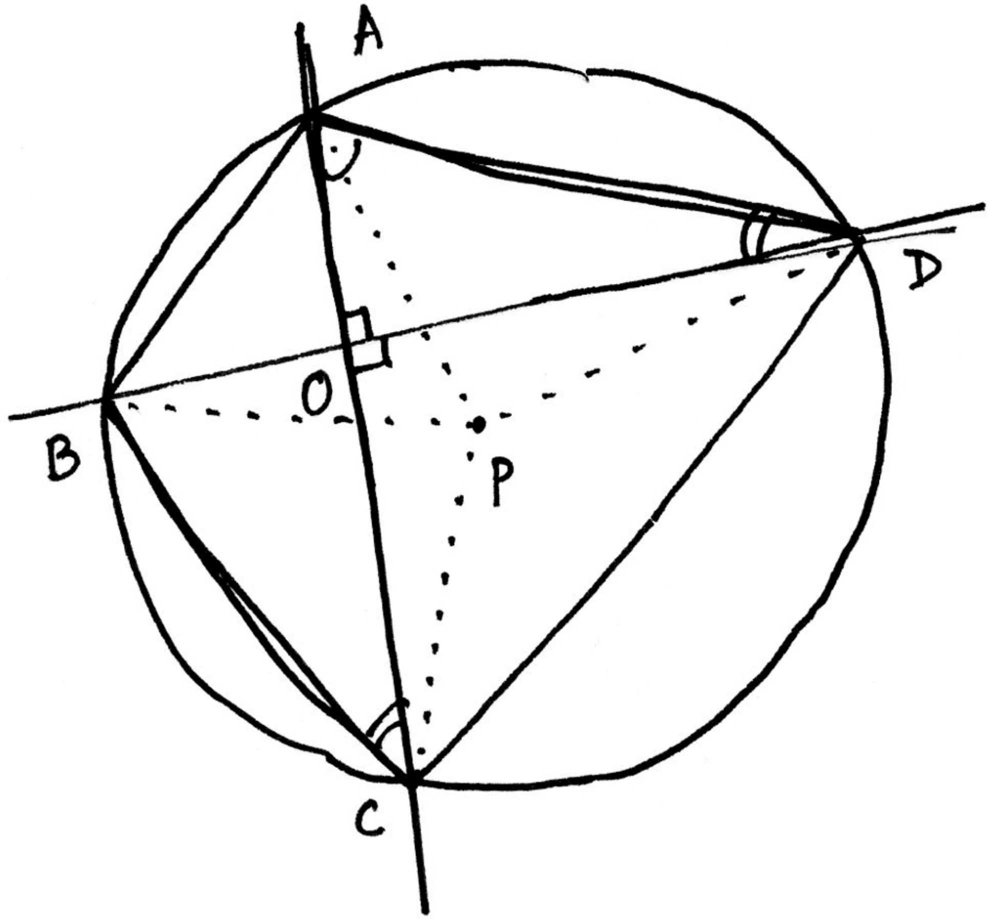


Imagine O is a fixed point and A moves on the circle with radius R .

Area of the slice swept by $OA = \int \frac{1}{2}(OA)^2 d\theta$

Sum of the areas of four even number slices :

$$
\begin{aligned}
\int_0^{\pi/4} \frac{1}{2}(OA)^2 d\theta + \frac{1}{2}(OB)^2 d\theta + \frac{1}{2}(OC)^2 d\theta + \frac{1}{2}(OD)^2 d\theta \\
&= \frac{1}{2}\int_0^{\pi/4}((OA)^2 + (OB)^2 + (OC)^2 + (OD)^2)d\theta \\
&= \frac{1}{2}\int_0^{\pi/4}(AB)^2 + (CD)^2 d\theta \\
&= \frac{1}{2}\int_0^{\pi/4} 4R^2 d\theta \\
&= \frac{\pi}{2}R^2
\end{aligned}
$$



Theorem. *In a circle with centre P and radius R. AC and BD are perpendicular to each other. Then $AB^2 + CD^2 = 4R^2$.*

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Proof. $\angle APB + \angle CPD = 2(\angle ACB + \angle CAD) = \pi$

Using the Cosine rule we have

$AB^2 + CD^2$
$= PA^2 + PB^2 + 2PA \cdot PB \cos(\angle APB) + PC^2 + PD^2 + 2PC.PD \cos(\angle CPD)$
$= 4R^2$

$\square$